Received: *December 7, 1999*, Accepted: *December 12, 1999*

Dynamics of anti-de Sitter domain walls

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Abstract: We study solutions corresponding to moving domain walls in the Randall-Sundrum universe. The bulk geometry is given by patching together black hole solutions in AdS_5, and the motion of the wall is determined from the junction equations. Observers on the wall interpret the motion as cosmological expansion or contraction. We describe the possible wall trajectories, and examine the consequences for localized gravity on the wall.

Keywords: Black Holes in String Theory, Cosmology of Theories beyond the SM.

JHEP12(1999)011

Contents

1. Introduction

Randall and Sundrum [1] have recently discovered a novel way of embedding four dimensional physics within a higher dimensional world. In their setup a four dimensional domain wall sits at a point in an infinite but highly curved fifth dimension, and a single normalizable zero mode of the gravitational field gives rise to newtonian gravity at large distances on the wall. The geometry as a whole is that of two regions of AdS_5 joined by the domain wall.

The presence of four dimensional Poincaré invariance requires a precise value for the domain wall tension. However, it is also of interest to consider non-Poincaré invariant solutions, both to better understand the mechanism of localized gravity and for possible cosmological applications. When the tension is not fine-tuned or there is additional matter on the wall, time dependent solutions typically result. Loosely speaking, such time dependence can come in two forms. As studied in [2]–[9], one can find cosmological solutions in which the bulk geometry is time dependent. Here we study the alternative case in which the bulk remains static but the domain wall acquires a velocity. Observers on the wall will interpret the motion of the wall through the static background as cosmological expansion or contraction. More generally, one can combine the two forms of time dependence to find a large class of solutions, but that will not be considered here. Of course, given any moving domain wall one can always transform coordinates to put it at rest, hence there is some overlap with the solutions found in [2]–[9] and those described here.

The equations of motion of the wall are found by a straightforward application of the thin wall formalism in general relativity [10], in which Einstein's equations are rewritten as junction conditions relating the discontinuity in the wall's extrinsic curvature to its energy-momentum tensor. We will consider the general solutions which have the symmetries of the standard Robertson-Walker geometries. Depending on the choice of parameters, we find bounded and unbounded wall trajectories, with both exponential and power law expansion in the latter case.

Our solutions are generally not described by regions of AdS_5 in the bulk, but rather by black hole solutions which reduce to AdS_5 as a non-extremality parameter is taken to zero. This has interesting implications for solutions in which the domain wall is slowly moving, so that one might expect to recover four dimensional gravity as in [1]. If taken to be too large, the event horizon of the black hole will render the zero mode nonnormalizable, and so destroy the effective four dimensional behavior. Small horizons result in zero modes, but the bulk solution has reduced symmetry compared to AdS_5. The lack of Lorentz invariance in the bulk manifests itself as a shift in the propagation speed of gravitational fluctuations on the wall, although the shift rapidly becomes negligible as the wall universe expands.

Eventually, one would like a microscopic description of the domain wall, either as a smooth solution of supergravity (along the lines of [11, 12, 13]) or as a fundamental brane of string theory. The present analysis is applicable to the former case in the thin wall limit. As an attempt to realize the latter we conclude by describing a configuration involving a spherical distribution of D3-branes, which ends up being unsuccessful due to the low value of the D3-brane tension.

Dynamical domain walls have also been studied recently in [14, 15].

2. Junction equations

The five dimensional bulk gravitational action is[1]

$$S = \frac{1}{16\pi G}\int_M d^5x\sqrt{-g}\left(R+\frac{12}{\ell^2}\right)+\frac{1}{8\pi G}\int_{\partial M} d^4x\sqrt{-\gamma}\, K\,. \tag{2.1}$$

We have allowed for a boundary ∂M with induced metric γ_{ab}, shortly to be identified with the domain wall. K is the trace of the extrinsic curvature of the boundary

$$K^{\mu\nu} = \nabla^\mu n^\nu\,, \tag{2.2}$$

where n^μ is the unit normal vector on ∂M. We are interested in patching two such regions together across a domain wall with four dimensional energy-momentum tensor T_{ab}. It is convenient to work in gaussian normal coordinates near the domain wall,

$$ds^2 = \gamma_{ab}dx^a dx^b + d\eta^2\,, \tag{2.3}$$

[1]Five dimensional indices are denoted by $\mu = 0,\ldots,4$, four dimensional indices on the domain wall by $a = 0,\ldots,3$, and spatial indices on the wall by $i = 1,\ldots,3$.

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with the wall at $\eta = 0$. Then Einstein's equations imply the junction conditions [10]:

$$\Delta K_{ab} = -8\pi G \left(T_{ab} - \frac{1}{3} T^c_c \gamma_{ab} \right), \tag{2.4}$$

where

$$\Delta K_{ab} = K^+_{ab} - K^-_{ab} = K_{ab}(\eta = \epsilon) - K_{ab}(\eta = -\epsilon) \,. \tag{2.5}$$

The relative minus sign arises because we have chosen the convention that n^μ points towards the region of increasing η. In the coordinates (2.3) the extrinsic curvature is

$$K_{ab} = \frac{1}{2} \partial_\eta \gamma_{ab} \,. \tag{2.6}$$

We refer to an "extremal" wall as one with energy-momentum tensor $T_{ab} = -\sigma \gamma_{ab}$, in which case the junction conditions become

$$\Delta K_{ab} = -\frac{8\pi G \sigma}{3} \gamma_{ab} \,. \tag{2.7}$$

We will consider bulk solutions that have the symmetries of flat, open, and closed Robertson-Walker universes. The unique solutions of Einstein's equations with the assumed properties are

$$ds^2 = -\left(k + \frac{r^2}{\ell^2} - \frac{\mu}{r^2} \right) dt^2 + r^2 d\Sigma_k^2 + \frac{dr^2}{(k + r^2/\ell^2 - \mu/r^2)} \,. \tag{2.8}$$

k takes the values 0, -1, $+1$, corresponding to flat, open, or closed geometries, and $d\Sigma_k^2$ is the corresponding metric on the unit three dimensional plane, hyperboloid, or sphere. The $k = -1$ solution is perhaps unfamiliar, but has been studied in [16, 17, 18]. When $\mu = 0$ the solutions are simply AdS_5 written in various coordinates, whereas $\mu \neq 0$ gives black hole solutions with horizons at $r = r_h$,

$$r_h^2 = \frac{\ell^2}{2} (-k + \sqrt{k^2 + 4\mu/\ell^2}) \,. \tag{2.9}$$

$k = 0, 1$ requires $\mu \geq 0$, and $k = -1$ requires $\mu \geq -\ell^2/4$. Note that the five dimensional version of Birkhoff's theorem requires that the solutions (2.8) be static.

The domain wall separating two such spacetimes (with the same k) is taken to be situated at $r = R(t)$, where $R(t)$ will be determined by solving the junction equations. As in the Randall-Sundrum geometry, r is taken to decrease as one moves away from the wall in *either* direction. One way to determine the junction equations is to transform to gaussian normal coordinates and use the formula (2.6). However, it is simpler to rewrite the equations in a coordinate independent form. Let u^μ be the velocity vector of the wall, $u^\mu u_\mu = -1$. Then the unit normal satisifies $n^\mu u_\mu = 0$, and we can rewrite (2.6) as

$$K_{ab} = \frac{1}{2} n^\mu \partial_\mu \gamma_{ab} \,. \tag{2.10}$$

Let us apply this to the metric

$$ds^2 = -f_k(r)dt^2 + r^2 d\Sigma_k^2 + f_k^{-1}(r)dr^2 \,. \tag{2.11}$$

We have $u^t = (f_k + \dot{R}^2)^{1/2} f_k^{-1}$, $u^r = \dot{R}$, where $\dot{R}$ is the derivative of R with respect to proper time τ. Then $n^t = -f_k^{-1}\dot{R}$, $n^r = -(f_k + \dot{R}^2)^{1/2}$. The minus sign arises because the coordinate r is decreasing in the direction n^μ. There are two nontrivial junction equations corresponding to the time and space components of (2.10). The spatial components of the extrinsic curvature are

$$K_{ij}^+ = -\frac{(f_k^+ + \dot{R}^2)^{1/2}}{R}\gamma_{ij} \,, \qquad K_{ij}^- = \frac{(f_k^- + \dot{R}^2)^{1/2}}{R}\gamma_{ij} \,, \tag{2.12}$$

wher $\pm$ denotes the two sides of the wall. The junction equation is then

$$\left[(f_k^+ + \dot{R}^2)^{1/2} + (f_k^- + \dot{R}^2)^{1/2}\right]\gamma_{ij} = 8\pi G R\left(T_{ij} - \frac{1}{3}T^a_a\gamma_{ij}\right), \tag{2.13}$$

or in the extremal case ($T_{ab} = -\sigma\gamma_{ab}$):

$$(f_k^+ + \dot{R}^2)^{1/2} + (f_k^- + \dot{R}^2)^{1/2} = \frac{8\pi G\sigma}{3}R \,. \tag{2.14}$$

It turns out that the junction equation for K_{tt} just gives the proper time derivative of the equations above, so we need not consider it further.

The junction equation determines $R(\tau)$, and so also the induced metric on the domain wall:

$$ds^2_{wall} = -d\tau^2 + R^2(\tau)d\Sigma_k^2 \,. \tag{2.15}$$

For reference, recall that in the standard FRW cosmology with energy density ρ, the scale factor R obeys the equation

$$\frac{1}{2}\dot{R}^2 - \frac{4\pi G_N \rho}{3}R^2 = -\frac{k}{2} \,. \tag{2.16}$$

2.1 Motion of extremal wall

We can rewrite (2.14) as the equation for a particle in a potential

$$\frac{1}{2}\dot{R}^2 + V(R) = -\frac{k}{2} \,, \tag{2.17}$$

with

$$V(R) = \frac{1}{2}\left(1 - \left(\frac{\sigma}{\sigma_c}\right)^2\right)\frac{R^2}{\ell^2} - \frac{1}{4}\frac{(\mu_+ + \mu_-)}{R^2} - \frac{1}{32}\left(\frac{\sigma}{\sigma_c}\right)^{-2}\frac{\ell^2(\mu_+ - \mu_-)^2}{R^6} \,, \tag{2.18}$$

where we have defined $\sigma_c \equiv 3/(4\pi G\ell)$. The Randall-Sundrum configuration results from taking $k = \mu_+ = \mu_- = 0$, and tuning the wall tension to be $\sigma = \sigma_c$. We now explore some of the possibilities that arise when we relax these conditions.

First consider the case $\mu_+ = \mu_- = 0$, in which the bulk geometries are regions of AdS_5. There are nine cases corresponding to the various values of σ and k:

$\sigma = \sigma_c$, $k = 0$: this gives the Randall-Sundrum configuration.

$\sigma = \sigma_c$, $k = -1$: $R(\tau) = |\tau|$. Wall passes between $R = 0$ and $R = \infty$, and crosses the coordinate horizon $r = \ell$ in finite proper time.

$\sigma = \sigma_c$, $k = +1$: no solution.

$\sigma > \sigma_c$, $k = 0$: $R = R_0 e^{\pm H\tau}$.

$\sigma > \sigma_c$, $k = -1$: $R = H^{-1} \sinh H\tau$.

$\sigma > \sigma_c$, $k = +1$: $R = H^{-1} \cosh H\tau$.

$\sigma < \sigma_c$, $k = 0$: no solution. [$\sigma < \sigma_c$, $k = -1$:] $R = H^{-1} \cos H\tau$.

$\sigma < \sigma_c$, $k = +1$: no solution.

Here $H = \frac{1}{\ell}\sqrt{|(\sigma/\sigma_c)^2 - 1|}$. In the three $\sigma > \sigma_c$ cases the wall metric is de-Sitter space, while in the $\sigma < \sigma_c$ case it is anti-de Sitter space. These solutions have appeared in different coordinates in the work of [2]–[9].

Now let us turn to the case where $\mu_+, \mu_- \neq 0$. The cases $\sigma > \sigma_c$, $\sigma < \sigma_c$ are qualitatively similar to those described above, either inflationary behavior for large R or bounded motion. Note, though, that the possibility of $\mu_+ + \mu_- < 0$ for $k = -1$ allows $V(R)$ to have nontrivial local maxima. The detailed forms of the trajectories can be found by integrating the equation of motion (2.17). We now consider the three $\sigma = \sigma_c$ cases:

$\sigma = \sigma_c$, $k = 0$: unbounded motion passing between $R = 0$ and $R(\tau) \approx 2^{1/4}(\mu_+ + \mu_-)^{1/4}|\tau|^{1/2}$. For large R the wall metric is that of a spatially flat radiation dominated cosmology.

$\sigma = \sigma_c$, $k = -1$: unbounded motion passing between $R = 0$ and $R(\tau) \approx |\tau|$.

$\sigma = \sigma_c$, $k = +1$: wall expands from $R = 0$ to maximum size, $V(R_{max}) = -1/2$, and recollapses.

2.2 Four dimensional description

It is interesting to consider the case $\sigma = \sigma_c$, $k = 0$, $\mu_+, \mu_- \neq 0$ in more detail. At late times, $\tau \gg (\mu_+ + \mu_-)^{1/2}$, the wall universe is slowly expanding and it becomes meaningful to ask whether, as in [1], conventional gravity in four approximately flat spacetime dimensions is recovered for distances large compared to ℓ but small compared to $(\dot{R}/R)^{-1}$. The latter condition means that we can take R to be constant over the time scale of interest. For simplicity, take $\mu_+ = \mu_- = \mu$, so that at late times the bulk geometry is that of the Randall-Sundrum configuration except that the bulk spacetime is the black hole geometry (2.8). There are two important new features: the infinite throat as $r \to 0$ has been replaced by an event horizon at $r = r_h = \ell^{1/2}\mu^{1/4}$, and four dimensional Lorentz invariance has been broken. To study the implications in a simplified setting we will replace the gravitational fluctuations with those of a massless bulk scalar field. When r_h is set to zero, the scalar field has an r independent normalizable zero mode, $\phi = \phi(t, \vec{x})$, which appears as a massless four dimensional scalar field on the domain wall. We can study the fate of the zero mode by examining the wave equation near the horizon using the coordinate $r_* = (\ell^2/4r_h)\ln((r-r_h)/\ell)$. Writing $\phi = e^{-i\omega t + i\vec{k}\cdot\vec{x}}\psi(r_*)$ the wave equation becomes

$$\left(\partial_{r_*}^2 + \omega^2 - \frac{4}{\ell r_h}e^{4r_h r_*/\ell^2}\vec{k}^2\right)\psi(r_*) = 0\,. \tag{2.19}$$

We see that there is no normalizable mode for $\omega \neq 0$, which seems to imply the lack of a massless field on the wall. On the other hand, we know that such a mode exists for $r_h = 0$ and we expect the limit $r_h \to 0$ to be smooth. The resolution is that for small r_h the geometry near the horizon is not reliable, and so we should impose a cutoff on the range of r. To implement this we work out the action of the candidate zero mode $\phi = \phi(t, \vec{x})$:

$$S = -\frac{1}{2}\int d^5x\,\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -\int d^4x\int_{r_c}^{R} dr\, r^3\left[-\frac{(\partial_t\phi)^2}{(r^2/\ell^2 - \mu/r^2)} + \frac{(\vec{\nabla}\phi)^2}{r^2}\right]. \tag{2.20}$$

Now, the trouble arises from attempting to take $r_c \to r_h$; instead, for small r_h we impose the cutoff $r_c = \ell$ corresponding to the region where trans-planckian curvatures begin to set in (we have in mind that $\ell \approx \ell_{\rm Pl}$). Then evaluating the integrals and expanding in r_h we find, assuming $R \gg \ell$:

$$S = -\frac{R^2}{2}\int d^4x\left[-\ell^2\left(1 + \frac{r_h^4}{\ell^2R^2}\right)(\partial_t\phi)^2 + (\vec{\nabla}\phi)^2\right]. \tag{2.21}$$

Rescaling ϕ and expressing the result in terms of the domain wall metric γ_{ab} we obtain:

$$S = -\frac{1}{2}\int d^4x\,\sqrt{-\gamma}\left[\left(1 + \frac{r_h^4}{\ell^2R^2}\right)\gamma^{tt}(\partial_t\phi)^2 + \gamma^{ij}\partial_i\phi\partial_j\phi\right], \tag{2.22}$$

which is the standard form except that the speed of light has been shifted to

$$c_{eff} \approx 1 - \frac{1}{2}\frac{r_h^4}{\ell^2R^2}\,. \tag{2.23}$$

We stress that this formula holds only when the correction term is small, and that the precise correction is not meaningful since it is sensitive to the position of the cutoff.

If the standard model fields live on the domain wall then their behavior is Lorentz invariant with the standard speed of light $c = 1$, whereas gravitational interactions (assuming that the scalar field results can be extrapolated to gravity) propagate at a slightly shifted speed due to the loss of Lorentz invariance in the bulk. In addition, we expect the tensor and momentum structure of the gravitational interactions to suffer small Lorentz violating corrections. As the universe expands, R becomes large and these effects rapidly become negligible.

Finally, in the case where r_h is large compared to ℓ, we expect the four dimensional description to be invalid due to the lack of a normalizable zero mode.

2.3 Matter on the wall

Now consider the case in which expansion results from matter on the domain wall. We take for the energy-momentum tensor

$$T_{ab} = -\sigma_c \gamma_{ab} + \rho u_a u_b + p(\gamma_{ab} + u_a u_b)\,, \tag{2.24}$$

corresponding to matter with energy density ρ and pressure p, in addition to the critical background wall tension σ_c. Energy conservation requires

$$\frac{d}{d\tau}(\rho R^3) = -p\frac{d}{d\tau}R^3\,. \tag{2.25}$$

The junction equation is found to be

$$(f_k^+ + \dot{R}^2)^{1/2} + (f_k^- + \dot{R}^2)^{1/2} = \frac{8\pi G(\sigma_c + \rho)}{3}R\,. \tag{2.26}$$

For $\rho \ll \sigma_c$ the corresponding potential is

$$V(R) = -\frac{\rho R^2}{\ell^2\sigma_c} - \frac{1}{4}\frac{(\mu_+ + \mu_-)}{R^2} - \frac{1}{32}\left(1 - \frac{2\rho}{\sigma_c}\right)\frac{\ell^2(\mu_+ - \mu_-)^2}{R^6}\,. \tag{2.27}$$

With $\mu_+ = \mu_- = 0$ the resulting potential is the same as that which arises in standard FRW cosmology (2.16), hence one recovers the conventional behavior. In particular, a $k = 0$ radiation dominated universe with $p = \rho/3$, $\rho = \rho_0/R^4$, leads to $R(\tau) = (4\rho_0/\ell^2\sigma_c)^{1/4}|\tau|^{1/2}$. A $k = 0$ matter dominated universe with $p = 0$, $\rho = \rho_0/R^3$ leads to $R(\tau) = (9\rho_0/2\ell^2\sigma_c)^{1/3}|\tau|^{2/3}$.

When $\mu_+, \mu_- \neq 0$, the late time behavior of a radiation dominated $k = 0$ universe again has $R(\tau) \sim \text{const.} \cdot |\tau|^{1/2}$, but with a shifted effective energy density: $\rho_0 \to \rho_0 + \ell^2\sigma_c(\mu_+ + \mu_-)/4$. On the other hand, the late time behavior of a matter dominated $k = 0$ universe is unchanged.

3. Attempt at a string theory realization

It is desirable to have an embedding of the Randall-Sundrum geometry into string theory. Here we briefly describe a largely unsuccessful attempt based on a spherical shell of D3-branes.[2] The construction fails because the tension of a D3-brane turns out to be too small by a factor of 3/2.

[2] See [19]–[23] for related discussions.

As is well known, the near horizon geometry of a collection of D3-branes is $AdS_5 \times S^5$,

$$ds^2 = -\frac{r^2}{\ell^2}dt^2 + r^2(d\vec{x})^2 + \frac{\ell^2}{r^2}dr^2 + \ell^2 d\Omega_5^2\,. \tag{3.1}$$

ℓ is related to the number of D3-branes by $\ell^4 = 4\pi g N(\alpha')^2$. In addition, there are N units of five-form flux present. Now, we attempt to patch two such regions with opposite five-form orientations together along a boundary of constant r. To satisfy charge conservation we need the boundary to carry $2N$ units of charge. We do this while preserving approximate SO(6) symmetry by distributing $2N$ D3-branes over the S^5. The branes are at coincident r positions, and their worldvolumes span $t, \vec{x}$. To preserve approximate SO(6) symmetry we require that the inter-brane spacing on S^5 be much smaller than the characteristic scale of the geometry ℓ. This requires $N \gg 1$.

Now, the tension of $2N$ D3-branes is, in terms of the five dimensional Newton constant,

$$\sigma = \frac{1}{2\pi G\ell} = \frac{2}{3}\sigma_c\,. \tag{3.2}$$

Hence the tension is too low to patch two such $AdS_5 \times S^5$ regions together in this manner. The only possibility is a time dependent $k = -1$ solution as discussed earlier. Such a solution collapses to a singularity in a time scale of order ℓ.

Acknowledgments

Supported by NSF Grant No. PHY-9600697. I have benefitted from discussions with J. Harvey, F. Larsen, E. Martinec, and R. Sundrum.

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